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THE EFFICIENT PROVISION OF REGIONAL PUBLIC GOODS IN THE PRESENCE OF BENEFIT SPILLOVERS AND POPULATION

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I. Introduction

The purpose of the present paper is to extend the analysis of public goods to situations where population is variable. The seminal contributions of Samuelson (1954 and 1955) to the pure theory of public goods are based upon the assumption of fixed population. This paper will present a model in which population is a variable to be endogenously determined. The context within which the model is developed is that of a federation of regional governments, although the results will have more general implications for population movement between the member units of a customs union and for other types of migration. The paper will also deal with the determination of the optimal level of production of public goods for a variable population when there are interjurisdictional benefit spillovers from the regional public goods.

At the heart of the Scott-Buchanan controversy concerning equalization payments to poor regions is the concept of efficiency in locational choice. The implicit argument of Buchanan was that a non-benefit tax system would create distortions in locational choice. The important point to be made is that fiscal distortion of the Buchanan type is the result of incorrect pricing of the public good as between jurisdictions. Efficient locational decisions require that differences in the comparative costs of regional public goods be reflected in their prices. If the tax prices of public goods enter into locational decisions then these decisions can be based on comparative costs only if prices reflect these costs, rather than the distribution of income or other irrelevant considerations. The present paper will assume that tax prices do in fact reflect the comparative costs of the public good and hence that interprovincial migration responds to the "correct" pricing signals. The main concern here is with the efficient provision of public goods by government when the size of the resident population responds to the level of public goods output.

It is assumed that the problem of preference revelation has been solved. In this spirit the paper is concerned primarily with defining efficiency conditions rather than with examining the institutional arrangements whereby they can be achieved.

In the next section a model will be developed to deal with population as a variable in a scheme for welfare optimization. In section III benefit spillovers will be introduced in a two-province federation. Section IV concludes the paper.

II. Regional Public Goods and Variable Population

The Tiebout model (1956) of locational choice presents a possible avenue for preference revelation with respect to public goods through the process of public good product differentiation. Different regions produce different public goods and people satisfy their public good wants by moving to the region that they prefer, thus revealing their preferences over the available possibility set. The model to be presented below follows the Tiebout framework in allowing individuals to choose between different combinations of public goods and tax prices.

In the rest of this section it will be assumed that the region upon which we concentrate is relatively small, in terms of population, in comparison with the other members of the federation. This enables us to assume that the relevant economic variables in the rest of the federation (taken as an entity) will remain unchanged by what happens in the region of interest.

There are two types of economic goods in our model, private goods and public goods. While differentiation of public goods is permitted between regions the private good is identical in every region. The private good is taken to be the numeraire in our analysis. If the price of private goods has been equalized throughout the region then the prices that are relevant for people contemplating a move from one region to another are factor prices and the tax prices of regional public goods. In what follows it will be assumed that all public goods are completely nonrivalrous within jurisdictions.[1] Activities of the central (federal) government will be ignored on the assumption that they affect all regions equally.[2]

The price of private goods is assumed the same everywhere but the prices of public goods will not in general be the same for every jurisdiction. This diversity of public good prices will apply both to the unit prices of regional (pure) public goods and to the individual tax prices. These prices will in general not be equalized unless individual tastes and factor endowments are the same between regions. Each regional jurisdiction produces only one public good. Apart from factor and product prices the quantity and characteristics of regional public goods are additional variables to be incorporated into locational decisions. People will be attracted into regions that produce public goods that they like. They will also be attracted to regions in which their factor services are highly valued and in which the tax price ofpublic goods is low. If factor prices are competitively determined and reflect true opportunity cost then the optimality of locational decisions will depend upon the extent to which tax prices are a reflection of the real cost of public goods.

In each province there is a public sector which produces a pure public good whose benefits and costs extend up to its jurisdictional borders and no further. The public sector is efficiently organized and behaves as if it were a perfectly competitive industry. For the province that is discussed here the output level of the provincial public good is denoted by R. The private sector in each province produces a homogeneous private good and its output level in the province of interest is denoted by X.

The population of the federation is mobile as between provinces and consists of workers with identical productive abilities. There are two factors of production, land (T) and labour (L). For simplicity it is assumed that the landlords are few in number and never change their province of residence.[3] Locational decisions are made on the basis of the following variables: (1) the output level and (2) characteristics of the various provincial public goods; (3) the tax price charged for these public goods; and (4) the wage rate paid in each region. While the wage rate paid by each industry (i.e. the private and public goods industries) in a given province will be the same the rate is likely to vary from one province to another. This variation will be partly attributable to differences in the provision of provincial public goods and in their unit costs.

Let us now be more specific and set out the provincial welfare function as

$$U = U(\frac{X}{N}, R), \tag{1}$$

where welfare is a function of the per capita consumption of private and public goods. This formulation is chosen because of the variability of provincial population. The setting for the model is a federation of provinces, where the population of the federation is fixed, but where interprovincial migration does take place. Maximization of social welfare for the federation requires maximization of per capita real income or welfare.

The production functions for private and public goods are as follows:

(2) $X = \bar{X}(L_X, T_X)$ and

(3) $R = \bar{R}(N - L_X, T_0 - T_X)$

where T_0 is the fixed amount of land in the province and N is the (variable) number of people (workers). These production functions are assumed to be linear homogeneous and twice differentiable.

The variability of population is introduced through a demand for residence function, which is given by

(4) $N = \bar{N}(W - \frac{PR}{N}, R)$,

where W ($W = \frac{\partial X}{\partial L_I} = P\frac{\partial R}{\partial N}$) is the wage rate and P is the price of the public good in terms of the numeraire private good. This price, P, is assumed for simplicity to remain constant.[4] Although P is assumed here to be constant it will not in general be the same for each province. The term P/N (= π) is the benefit tax price per person for the provincial public good and $W - \frac{PR}{N}$ is the wage rate net of tax. The supply of labour offered by each worker is assumed to remain constant throughout the analysis. In what follows subscripts will be used on U, $\widetilde{R}$, $\widetilde{X}$, and $\widetilde{N}$ to denote partial derivatives, as for example $\frac{\partial \widetilde{R}}{\partial L_1} = \widetilde{R}_1$. The partial derivatives $\widetilde{N}_1$ and $\widetilde{N}_2$ are assumed to be positive.

Since the regional welfare function is to be maximized subject to the conditions imposed by (2), (3) and (4) we may write the following Lagrangian expression:

$$(5)\quad M = U(\tfrac{X}{N},R) + \lambda_1[X-\widetilde{X}(L_X,T_X)]$$

$$+ \lambda_2[R-\widetilde{R}(N-L_X,T_0-T_X)]$$

$$+ \lambda_3[N-\widetilde{N}(\tfrac{P\partial R}{\partial N} - \tfrac{PR}{N}, R)]$$

From this expression the following first order conditions for welfare maximization may be derived:

$$(6)\quad \frac{\partial M}{\partial X} = \frac{U_1}{N} + \lambda_1 = 0 ,$$

$$(7)\quad \frac{\partial M}{\partial R} = U_2 + \lambda_2 + \lambda_3 \ (\widetilde{N}_1\frac{P}{N} - \widetilde{N}_2) = 0 ,$$

$$(8)\quad \frac{\partial M}{\partial L_X} = -\lambda_1\widetilde{X}_1 + \lambda_2\widetilde{R}_1 = 0 ,$$

$$(9)\quad \frac{\partial M}{\partial T_X} = -\lambda_1\widetilde{X}_2 + \lambda_2\widetilde{R}_2 = 0 , \quad \text{and}$$

$$(10)\quad \frac{\partial M}{\partial N} = - \frac{X}{N^2}U_1 - \lambda_2\widetilde{R}_1 + \lambda_3 - \lambda_3\widetilde{N}_1 P\widetilde{R}_{11} - \lambda_3\widetilde{N}_1\frac{PR}{N^2} = 0.$$

It is possible to solve for λ_1, λ_2 and λ_3 from (6), (7) and (8):

$$(11) \quad \lambda_1 = -\frac{U_1}{N},$$

$$(12) \quad \lambda_2 = -U_1\frac{P}{N}, \quad \text{and}$$

$$(13) \quad \lambda_3 = \frac{NU_2 - U_1 P}{N\bar{N}_2 - \bar{N}_1 P}.$$

If we substitute these values into (1) then we obtain:

$$(14) \quad -\frac{X}{N^2}U_1 + U_1\frac{P}{N}\bar{R}_1$$

$$+ \left(\frac{NU_2 - U_1 P}{N\bar{N}_2 - \bar{N}_1 P}\right)\left(1 - \bar{N}_1 P\bar{R}_{11} - \bar{N}_1 P\frac{R}{N^2}\right) = 0$$

or

$$(15) \quad \left(\frac{X}{N} - \frac{\partial \bar{X}}{\partial N}\right)\left(\bar{N}_2 - \bar{N}_1\frac{P}{N}\right) = (NS - P)\left(1 - \bar{N}_1\bar{R}_{11}P - \bar{N}_1\frac{PR}{N^2}\right)$$

From (15) it is clear that when $\bar{N}_1 = \bar{N}_2 = 0$ then we have the Samuelson condition $NS=P$, where S is the marginal rate of substitution of private for public goods.

Let us examine the individual term in (15) to determine whether the signs of the bracketed expressions may be found. The first such expression may be rewritten $\left(\frac{X}{L_1+L_2} - \frac{\partial \bar{X}}{\partial L_1}\right)$ and this expression would be unambiguously negative if the marginal product of labour were constant or increasing:

$\frac{\partial \widetilde{X}}{\partial L_1} \geq \frac{X}{L_1} > \frac{X}{L_1+L_2}$. However, if we assume that it falls as more labour is added then the sign of $(\frac{X}{N} - \widetilde{X}_1)$ is indeterminate. Now let us take the total differential of (4):

$$(16) \quad dN(1-\widetilde{N}_1\widetilde{R}_{11}P-\widetilde{N}_1\frac{PR}{N^2}) = (\widetilde{N}_2-\widetilde{N}_1\frac{P}{N})dR$$

so that we may rewrite (15) as

$$(17) \quad (\frac{X}{W} - \widetilde{X}_1)\frac{dN}{dR} = (NS-P).$$

However, in spite of the *a priori* positive signs on $\widetilde{N}_1$ and $\widetilde{N}_2$ we are unable to get a determinate sign for $\frac{dN}{dR}$, where

$$\frac{dN}{dR} = (\widetilde{N}_2-\widetilde{N}_1\frac{P}{N})/(1-\widetilde{N}_1PR_{11}-\widetilde{N}_1\frac{PR}{N^2}) \ .$$

If strongly diminishing returns to labour have not set in and hence $(\frac{X}{N} - \widetilde{X}_1)<0$ and if $\frac{dN}{dR} > 0$ then optimality requires $NS<P$ and $S<\frac{P}{N} = \pi$. This means that the Samuelson first order condition for optimality, which requires that $NS=P$, is applicable only when $\frac{dN}{dR} = 0$, $(\frac{X}{N} - \widetilde{X}_1) = 0$, or when both of these terms take zero values. When the left-hand side of (17) is negative then "overproduction" of the public good is called for. A welfare maximizing government will expand the output of public goods beyond the point where $NS=P$.

III. Benefit Spillovers and Population Mobility

In this section the analysis of the previous section is generalized to incorporate benefit spillovers in a two-province federation. Again each province produces a homogeneous private good and a single public good. The population of province I will be denoted by N and that of province 2 by M while the fixed population of the federation is given by N_0. The welfare function for province I is given by (18) while (19) gives the corresponding welfare function for province 2:

(18) $U = U(\frac{X}{N}, \varsigma_{11}R_1+\varsigma_{21}R_2)$, and

(19) $W = W(\frac{Z}{M}, \varsigma_{22}R_2+\varsigma_{12}R_1)$.

In (18) and (19) ς_{ij} [5] gives the spillover of benefits for R_i, produced in province i, to province j. Note that we have assumed that provincial public goods are completely non-rivalrous within a province but we make no assumption concerning interprovincial rivalry in consumption.[6] In (19) private goods consumption is denoted by Z.

The production functions are given as follows:

(20) $X = \bar{X}(N_1,T_1)$,

(21) $R_1 = \bar{R}_1(N-N_1,T_0-T_1)$,

(22) $Z = \bar{Z}(M_1,K_1)$, and

$$(23) \quad R_2 = \bar{R}_2\,(M-M_1,\ K_0-K_1)$$

As in the previous section, landlords are assumed to be immobile. They are assumed to own T_0 units of land in province 1 and K_0 units in province 2.

The demand for residence function in province 1 is given by:

$$(24) \quad N = \bar{N}\left(\frac{\partial X}{\partial N} - \frac{P_1R_1}{N} - \frac{\partial \bar{Z}}{\partial M} + \frac{P_2R_2}{M},\ \zeta_{11}R_1+\zeta_{21}R_2,\ \zeta_{22}R_2+\zeta_{12}R_1\right).$$

The first argument of $\bar{N}$ is the difference between the wage rate net of tax in the two provinces. The second argument in $\bar{N}$ is the level of public good consumption (measured in units of R_1) available in province 1 while the third argument of $\bar{N}$ refers to the corresponding availability of public goods in province 2. The final population equation is given by (25):

$$(25) \quad N + M = N_0 \ .$$

The welfare maximization problem for the federation may be set up in the form of the following Lagrangian expression:

$$(26) \quad V = U\left(\frac{X}{N},\ \zeta_{11}R_1+\zeta_{21}R_2\right) + \lambda_1\left[W\left(\frac{Z}{M},\ \zeta_{22}R_2+\zeta_{12}R_1\right)-W^0\right]$$
$$+ \lambda_2[X-\bar{X}(N_1,T_1)] + \lambda_3[R_1-\bar{R}_1(N-N_1,T_0-T_1)]$$
$$+ \lambda_4[Z-\bar{Z}(M_1,K_1)] + \lambda_5[R_2-\bar{R}_2(M-M_1,K_0-K_1)]$$

$$+ \lambda_6[N-\widetilde{N}(\frac{\partial\widetilde{X}}{\partial N_1} - \frac{P_1R_1}{N} - \frac{\partial Z}{\partial M} + \frac{P_2R_2}{M}, \zeta_{11}R_1+\zeta_{21}R_2,$$

$$\zeta_{22}R_2+\zeta_{21}R_1)] + \lambda_7[N_0-N-M]$$

The first order conditions for welfare maximization are as follows:

$$(27) \quad \frac{\partial V}{\partial X} = \frac{U_1}{N} + \lambda_2 = 0 \;,$$

$$(28) \quad \frac{\partial V}{\partial Z} = \lambda_1\frac{W_1}{M} + \lambda_4 = 0 \;,$$

$$(29) \quad \frac{\partial V}{\partial R_1} = \zeta_{11}U_2+\lambda_1\zeta_{12}W_2+\lambda_3+\lambda_6\widetilde{N}_1\frac{P_1}{N} - \lambda_6\widetilde{N}_2\zeta_{11}-\lambda_6\widetilde{N}_3\zeta_{12} = 0$$

$$(30) \quad \frac{\partial V}{\partial R_2} = \zeta_{21}U_2+\lambda_1\zeta_{22}W_2+\lambda_5-\lambda_6\widetilde{N}_1\frac{P_2}{M} - \lambda_6\widetilde{N}_2\zeta_{21}-\lambda_6\widetilde{N}_3\zeta_{22} = 0$$

$$(31) \quad \frac{\partial V}{\partial N_1} = -\lambda_2\frac{\partial\widetilde{X}}{\partial N_1} + \lambda_3\frac{\partial\widetilde{R}}{\partial N_1} = 0$$

$$(32) \quad \frac{\partial V}{\partial T_1} = -\lambda_2\frac{\partial\widetilde{X}}{\partial T_1} + \lambda_3\frac{\partial\widetilde{R}}{\partial T_1} = 0$$

$$(33) \quad \frac{\partial V}{\partial M_1} = -\lambda_4\frac{\partial\widetilde{Z}}{\partial M_1} + \lambda_5\frac{\partial\widetilde{R}}{\partial M_1} = 0$$

$$(34) \quad \frac{\partial V}{\partial K_1} = -\lambda_4\frac{\partial\widetilde{Z}}{\partial K_1} + \lambda_5\frac{\partial\widetilde{R}_2}{\partial K_1} = 0$$

$$(35) \quad \frac{\partial V}{\partial N} = -\frac{X}{N^2}U_1 - \lambda_3\frac{\partial\widetilde{R}_1}{\partial N} + \lambda_6-\lambda_6\frac{\widetilde{N}_1P_1R_1}{N^2} - \widetilde{N}_1\lambda_6\widetilde{X}_{11} - \lambda_7 = 0$$

$$(36) \quad \frac{\partial V}{\partial M} = -\lambda_1\frac{Z}{M^2}W_1-\lambda_5\frac{\partial\widetilde{R}_2}{\partial M} + \lambda_6\widetilde{N}_1\widetilde{Z}_{11} + \lambda_6\widetilde{N}_1\frac{P_2R_2}{M^2} - \lambda_7 = 0 \;.$$

Before solving these equations let us find the total differential of (24):

$$(37)\quad dN = \bar{N}_1(\bar{X}_{11}dN - \frac{P_1}{N}dR_1 + \frac{P_1R_1}{N^2}dN + \bar{Z}_{11}dN$$

$$+ P_2\frac{dR_2}{M} + \frac{P_2R_2}{M^2}dN) + \bar{N}_2(\zeta_{11}dR_1 + \zeta_{21}dR_2)$$

$$+ \bar{N}_3(\zeta_{22}dR_2 + \zeta_{12}dR_1).$$

From this expression we may find

$$(38)\quad dN(1-\bar{N}_1X_{11} - \frac{P_1R_1}{N^2}\bar{N}_1 - Z_{11}\bar{N}_1 - \bar{N}_1\frac{P_2R_2}{M^2})$$

$$= dR_1(\bar{N}_2\zeta_{11} + \bar{N}_3\zeta_{12} + \frac{P_1}{N}N_1) + dR_2(\bar{N}_2\zeta_{21} + \bar{N}_3\zeta_{22} + N_1\frac{P_2}{M}) .$$

Rewrite (38) as

$$(39)\quad H_3dN = H_1dR_1 + H_2dR_2 .$$

These manipulations enable us to rewrite (29) and (30) as:

$$(40)\quad \zeta_{11}U_2 + \lambda_1\zeta_{12}W_2 + \lambda_3 - H_1\lambda_6 = 0, \text{ and}$$

$$(41)\quad \zeta_{21}U_2 + \lambda_1\zeta_{22}W_2 + \lambda_5 - H_2\lambda_6 = 0.$$

From (35) and (36) we have

$$(42)\quad -\frac{X}{N^2}U_1 + \lambda_1\frac{Z}{M^2}W_1 - \lambda_3\frac{\partial\bar{R}_1}{\partial N} + \lambda_5\frac{\partial\bar{R}}{\partial M}$$

$$+ \lambda_6(1-\bar{N}_1\bar{X}_{11} - \bar{N}\frac{P_1R_1}{N^2} - \bar{Z}_{11}\bar{N}_1 - \bar{N}_1\frac{P_2R_2}{M^2}) = 0$$

and (42) may be written

$$(43)\quad -\frac{X}{N^2}U_1+\lambda_1\frac{Z}{M^2}W_1-\lambda_3\frac{\partial\widetilde{R}_1}{\partial N}+\lambda_5\frac{\partial\widetilde{R}_2}{\partial M}+H_3\lambda_6=0.$$

Now if we assume that workers are employed efficiently then $\frac{\partial X}{\partial N_1}=-P_1\frac{\partial R_1}{\partial N_1}$ and $\frac{\partial Z}{\partial M_1}=-P_2\frac{\partial R_2}{\partial M_1}$. This enables us to derive $+P_1\lambda_2=\lambda_3$ and $+P_2\lambda_4=\lambda_5$ from (31) and (33). Next rewrite (43) as (44)

$$(44)\quad -\frac{X}{N^2}U_1-P_1\lambda_2\frac{\partial\widetilde{R}_1}{\partial N}+\lambda_1\frac{Z}{M^2}W_1+P_2\lambda_4\frac{\partial\widetilde{R}_2}{\partial M}=-H_3\lambda_6$$

or

$$\frac{U_1}{N}\frac{A}{H_3}-\frac{W_1}{M}\frac{B}{H_3}\lambda_1=\lambda_6$$

where $A=\frac{X}{N}-\frac{\partial X}{\partial N_1}$ and

$$B=\frac{Z}{M}-\frac{\partial X}{\partial M_1}$$

Substitute into (40) and (41)

$$(45)\quad \zeta_{11}U_2+\lambda_1\zeta_{12}W_2=P_1\frac{U_1}{N}+\left(\frac{U_1}{N}A-\frac{W_1B}{M}\lambda_1\right)\frac{dN}{dR_1}$$

$$(46)\quad \zeta_{21}U_2+\lambda_1\zeta_{22}W_2=P_2\frac{W_1}{M}\lambda_1+\left(\frac{U_1}{N}A-\frac{W_1}{M}B\lambda_1\right)\frac{dN}{dR_2}\ .$$

By our assumption that the private good is the numeraire of the system we have $\lambda_2=\lambda_4$ so that, from (27) $\lambda_1=\frac{U_1}{N}\frac{M}{W_1}$.

This enables us to rewrite (45) and (46) as

$$(47)\quad \zeta_{11}NS_1+\zeta_{12}MS_2 = P_1+(A-B)\frac{dN}{dR_1}$$

and

$$(48)\quad \zeta_{21}NS_1+\zeta_{22}MS_2 = P_2+(A-B)\frac{dN}{dR_2}$$

The two equations (47) and (48) present the first order optimality conditions for the production of the two provincial public goods.[7] If $\frac{dN}{dR_1} = \frac{dN}{dR_2} = 0$ then these equations replicate the familiar Samuelsonian conditions for optimality. Let us provisionally assume $\frac{dN}{dR_1} = \frac{dN}{dR_2} = 0$ and let each province be assumed to optimize independently of the other. This means that each region will equate its valuation on its own public good to the unit cost of that good so that (49) and (50) will obtain:

$$(49)\quad \zeta_{11}NS_1 = P_1 \quad \text{and}$$

$$(50)\quad \zeta_{22}MS_2 = P_2 .$$

The federal government may attempt to optimize upon this situation by paying a unit subsidy of V_1 and V_2 to provinces 1 and 2, respectively. These subsidies may be calculated as follows:

$$(51)\quad \zeta_{11}NS_1 = P_1-\zeta_{12}MS_2 = P_1-V_1$$

and

(52) $\varsigma_{22}MS_2 = P_2-\varsigma_{21}NS_1 = P_2-V_2$.

Equations (47) and (48) may be solved simultaneously for the values of NS_1 and MS_2 to yield:

(53) $$V_1 = \delta_{12}MS_2 = \frac{\varsigma_{12}(P_2\varsigma_{11}-P_1\varsigma_{21})}{|\varsigma|}$$

and

(54) $$V_2 = \varsigma_{21}NS_1 = \frac{\varsigma_{21}(P_1\varsigma_{22}-P_2\varsigma_{12})}{|\varsigma|}$$

where $|\varsigma| = \varsigma_{11}\varsigma_{22}-\varsigma_{12}\varsigma_{21}$.

This analysis informs us that if population is exogenously determined then optimizing subsidies may be calculated on the basis of spillover coefficients and the unit cost of each of the public goods.

Let us now drop the assumption of exogenous determination of the levels of provincial population and assume that $\frac{dN}{dR_1} > 0$ and $\frac{dN}{dR_2} < 0$ in (47) and (48).[8] In order to devise a system of optimizing subsidies it is necessary to find out whether or not the provinces adjust public output to reflect induced population changes. If they do then they will independently achieve equilibrium positions characterized by $NS_1 = P_1+(A-B)\frac{dN}{dR_1}$ and

$$MS_2 = P_2+(A-B)\frac{dN}{dR_2} .$$

Let

$$D_1 = P_1 + (A-B)\frac{dN}{dR_1} \quad \text{and}$$

$$D_2 = P_2 + (A-B)\frac{dN}{dR_2} \quad \text{so}$$

that the optimizing subsidies become

$$(55) \quad V_1{}^I = \frac{\varsigma_{12}}{|\varsigma|}(D_2\varsigma_{11} - D_1\varsigma_{21}) \quad \text{and}$$

$$(56) \quad V_2{}^I = \frac{\varsigma_{21}}{|\varsigma|}(D_1\varsigma_{22} - D_2\varsigma_{12}) \ .$$

Note that our original assumptions of the constancy of P_1 and P_2 are no longer sufficient to ensure that D_1 and D_2 will be constants. In general the subsidy system must now be much more sensitive to the level of output of the provincial public goods.[9] An essential non-linearity has been introduced in the subsidy system and this non-linearity results from the possible non-linearity of $\frac{dN}{dR_1}$ and $\frac{dN}{dR_2}$ and of $A-B (= \frac{X}{N} - \frac{\partial X}{\partial N} - \frac{Z}{M} + \frac{\partial Z}{\partial M})$. Equations (55) and (56) may be rewritten as follows:

$$(57) \quad V_1{}^I = \frac{\varsigma_{12}}{|\varsigma|}[(P_2\varsigma_{11} - P_1\varsigma_{21}) + (A-B)(\frac{dN}{dR_2}\varsigma_{11} - \frac{dN}{dR_1}\varsigma_{21})]$$

and

$$(58) \quad V_2{}^I = \frac{\varsigma_{21}}{|\varsigma|}[(P_1\varsigma_{22} - P_2\varsigma_{12}) + (A-B)(\frac{dN}{dR_1}\varsigma_{22} - \frac{dN}{dR_2}\varsigma_{12})].$$

If we retain our provisional assumptions that $\frac{dN}{dR_2} < 0$ and $\frac{dN}{dR_1} > 0$ then the expression $(\varsigma_{11}\frac{dN}{dR_2} - \varsigma_{21}\frac{dN}{dR_1})$, in (57), will be negative while $(\varsigma_{22}\frac{dN}{dR_1} - \varsigma_{12}\frac{dN}{dR_2})$ will be positive. If (A-B) is positive then the non-zero "mobility" terms $(\frac{dN}{dR_2}$ and $\frac{dN}{dR_1})$ will have the effect of increasing the unit subsidy to province 2 and they will reduce the unit subsidy to province 1.

If the production functions in our model are assumed to be linearly homogeneous then we may write:

(59) $A = \frac{X}{N} - \frac{\partial X}{\partial N} = \frac{T_1}{N}\frac{\partial X}{\partial T_1}$ which is the per capita rent on land used in the private goods industry. This enables us to consider A as a measure of diminishing returns to labour in province 1, while B is similarly defined for province 2. Hence the optimizing subsidy to province 1 will be lower than it would have been without the locational effects associated with the two provincial public goods. By the same reasoning the optimizing subsidy to province 2 will be larger.

Now let us consider the alternative behavioural assumption that the provincial governments select an optimizing level of public goods production without regard for the induced effects on resident population. This means that their respective equilibrium positions will be given by:

(60) $\varsigma_{11}NS_1 = P_1$

and

(61) $\varsigma_{22} MS_2 = P_2$

The optimizing subsidies will be as follows:

(62) $V_1^{II} = \varsigma_{12} MS_2 - (A-B)\frac{dN}{dR_1}$ and

(63) $V_2^{II} = \varsigma_{21} NS_1 - (A-B)\frac{dN}{dR_2}$.

If we solve (47) and (48) for NS_1 and MS_2 then by substitution into (63) and (62) we have:

(64) $$V_1^{II} = \frac{\varsigma_{12}}{|\varsigma|}[(P_2\varsigma_{11} - P_1\varsigma_{21}) + (A-B)(\varsigma_{11}\frac{dN}{dR_2} - \varsigma_{21}\frac{dN}{dR_1})]$$

$$- (A-B)\frac{dN}{dR_1} .$$

or

(65) $$V_1^{II} = V_1 + (A-B)[\frac{\varsigma_{12}}{|\varsigma|}(\varsigma_{11}\frac{dN}{dR_2} - \varsigma_{21}\frac{dN}{dR_1}) - \frac{dN}{dR_1}]$$

$$= V_1^{I} - (A-B)\frac{dN}{dR_1} \quad \text{and}$$

(66) $$V_2^{II} = \frac{\varsigma_{21}}{|\varsigma|}[P_1\varsigma_{22} - P_2\varsigma_{12} + (A-B)(\varsigma_{22}\frac{dN}{dR_1} - \varsigma_{12}\frac{dN}{dR_2})$$

$$- (A-B)\frac{dN}{dR_2}$$

or

$$(67)\quad V_2{}^{II} = V_2+(A-B)\left[\frac{\zeta_{21}}{|\zeta|}\left(\zeta_{22}\frac{dN}{dR_1} - \zeta_{12}\frac{dN}{dR_2}\right) - \frac{dN}{dR_2}\right]$$

$$= V_2{}^{I} - (A-B)\frac{dN}{dR_2}\ .$$

Equations (65) and (67) are very informative as to the changes in federal optimization policy that are wrought by recognition of the locational effects of public goods. Assume the following:

(a) $(A-B) > 0$,

(b) $\frac{dN}{dR_1} > 0$,

(c) $\frac{dN}{dR_2} < 0$

(d) $V_1 > 0$ and

(e) $|\zeta| > 0$

This means that $V_1{}^{I} < V_1$ and that $V_1{}^{II} < V_1{}^{I}$. The presence of spillover benefits makes it efficient for province 1 to expand its output of public goods and it is for this reason that $V_1 > 0$. However, the increase in R_1 has the effect of reducing the marginal product of labour, since it attracts new residents into the province. To adjust for this effect the optimal unit subsidy ought to be less than V_1. As has been shown above the subsidy should be $V_1{}^{I}$ or $V_1{}^{II}$, depending upon the behavioural assumptions that are made. If the provincial governments ignore the spillover benefits that they create but do in fact recognize that an increase in their public good output will attract new residents then $V_1{}^{I}$ and $V_2{}^{I}$ are the appropriate federal subsidies. This is the situation that obtains when the equality $NS_1 = P_1+(A-B)\frac{dN}{dR_1}$ is achieved independently of federal subsidies. When the government of province 1 regards the term $(A-B)\frac{dN}{dR_1}$ as part of the marginal cost associated with its public good then the optimizing subsidy to the province is given by $V_1{}^{I}$. When the government fails to incorporate this mobility factor in its decision process then the corrective subsidy becomes $V_1{}^{II}$. By our illustrative assumptions on the signs of the relevant variables it turns out that $V_1{}^{II} < V_1{}^{I}$ and the difference between the two is given by $(A-B)\frac{dN}{dR_1}$. It should be noted again that $V_1{}^{I}$ and $V_1{}^{II}$ (as well as $V_2{}^{I}$ and $V_2{}^{II}$) cannot in general be expected to remain constant at different levels of R_1 (and R_2). Furthermore, V_1 or V_2 or both may be zero while $V_1{}^{I}$, $V_2{}^{I}$, $V_1{}^{II}$ and $V_2{}^{II}$ (or any combination of these) are non-zero. The most obvious case where this is likely to be true is that in which there are no direct spillover benefits (i.e. $\zeta_{12} = \zeta_{21} = 0$).

IV. Conclusion

Introduction of endogenously determined population into the analysis of regional public goods has been shown to modify the optimality conditions. Regional welfare functions have been formulated in terms of the per capita consumption of private and public goods in order to facilitate the treatment of interprovincial migration within a federation. "Mobility functions" are also introduced to relate public goods and factor prices to interprovincial population movements.

Optimizing federal subsidies have been derived in order to correct for benefit spillovers between provinces. Federal optimization policy is achieved by adjustment in the unit costs incurred by provincial governments in the production of provincial public goods. These conditional grants are financed through some unspecified lump sum federal taxes which create no locational bias. The subsidies will depend not only upon benefit spillovers and the unit costs of producing public goods but also upon induced "population pressure on the land". This refers to the effects of changes in the output of public goods upon the allocation of population between provinces and the consequent impact on labour productivity.

The analysis of section III treated benefit spillovers for a two-province federation with interprovincial population mobility. The two-good model could be extended to n provinces by introducing (n-1) mobility functions without lack of generality. Other extensions could be made by introducing additional factors of production and other private[10] and public goods. The two-factor and two-good model that has been used as the basis for the present analysis is the traditional framework for international trade theory. Trade theory places considerable attention on differences in the factor intensities of different final products. This issue has been avoided here through the assumption of constant prices. Relaxation of the assumption of equal factor intensities will be undertaken in another paper.

FOOTNOTES

1. This enables us to ignore the problem of congestion costs associated with partially rivalrous public goods.

2. An important exception to our general policy with respect to activities of the federal government concerns federal subsidies to the provinces. It will not in general be found that optimizing subsidies are the same for each provincial government.

3. In what follows little attention is paid to these landlords. The provincial population N will typically refer to workers and landlords. Since our main concern is with the mobile workers (and not with the immobile landlords) we have not bothered to write $\bar{N}=\ell+t$, where ℓ is the number of workers and t is the number of landlords.

   An alternative treatment of the earnings of landlords would have been to invest each worker-resident of the province with equal property rights in land. This would enable us to avoid distributional problems associated with the provincial welfare function given in (1) below.

4. The assumption that P is constant may be achieved by positing that factor intensities are the same in both the private and public goods industries.

5. See Vardy (1971, pp. 13-27) for a more detailed discussion of spillover coefficients.

6. This means that our analysis is equally applicable to rivalrous interprovincial spillovers as to non-rivalrous spillovers.

7. Note that it has not been possible to assign a priori signs to $\frac{H_1}{N_3}$ or to $\frac{H_2}{H_3}$. An appeal to second order conditions was found to yield rather unwieldy results.

8. See footnote 7 and note that $\frac{dN}{dR_1} = \frac{H_1}{H_3}$ and $\frac{dN}{dR_2} = \frac{H_2}{H_3}$.

9. This results from diminishing returns to labour reflected in A and B as well as from the non-linearity

of $\frac{dN}{dR_1}$ and $\frac{dN}{dR_2}$. Our assumption of equal factor intensities in private and public good production (in footnote 4) was responsible for the constancy of P_1 and P_2 but is not sufficient to yield constancy in D_1 and D_2.

(10) Also, an extension of the basic models of sections II and III would gain in rigour through the introduction of a third good. This third good would be a private good and both private goods would be traded. Each province would export one private good and import the others. The main advantage of doing this, for our purposes, would be that it would provide a less artificial device for the introduction of the numeraire. One of the traded goods would be the numeraire and, with free trade, this good would be equally available in all regions. The extension to two private goods would not affect our basic results but would provide us with an unambiguous numeraire. Such an extension has not been presented since it would make our analysis more cumbersome.

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